

HOW TO CONSTRUCT THE SYMMETRIC CYCLE OF LENGTH 5 USING HAJÓS CONSTRUCTION

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A vertex coloring of a digraph is acyclic if there are no monochromatic directed cycles. The dichromatic number of a digraph D was introduced by Neumann-Lara in 1982 [3] as the minimum number of colors of an acyclic coloring of D , denoted by $dc(D)$. The dichromatic number of a digraph is an extension of the chromatic number of a graph and several concepts and results for the chromatic number of a graph have been extended to digraphs using the dichromatic number. A digraph D is **r -critical** if $dc(D) = r$ and $dc(H) < r$ for every proper subdigraph H of D . In 2020 Bang-Jensen et al. [1] extended the well-known Hajós construction for graphs to digraphs: any r -critical digraph can be obtained by Hajós constructions using complete symmetric digraphs on r vertices, $D(K_r)$. Although the result was proved, it is not a trivial task to obtain even simple digraphs such as symmetric cycles of odd length. In particular, the authors left as an open problem how to construct the symmetric cycle $D(C_5)$ using directed Hajós joins and identifying non-adjacent vertices.

We adapted a rank-based genetic algorithm to solve this problem by the introduction of innovative recombination and mutation operators from graph theory [2]. This algorithm obtained a sequence of Hajós operations that went through more than 5000 generations. In this talk, we present the reduced sequence of 16 Hajós operations to construct the symmetric cycle of order 5 and how to generalize this sequence to obtain any odd symmetric cycle.

References

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