ON SOME GENERALIZED VERTEX FOLKMAN NUMBERS

Zohair Raza Hassan

Department of Computer Science, Rochester Institute of Technology, USA
e-mail: zh5337@rit.edu

Yu Jiang

College of Electronics and Information Engineering, Beibu Gulf University, China
e-mail: wdjiangyu@126.com

David E. Narváez

Computer Science Department, University of Rochester, USA
e-mail: david.narvaez@rochester.edu

Stanisław P. Radziszowski

Department of Computer Science, Rochester Institute of Technology, USA
e-mail: spr@cs.rit.edu

Xiaodong Xu

Guangxi Academy of Sciences, Nanning, China
e-mail: xxdmaths@sina.com

For a graph G and integers $a_i \geq 1$, the expression $G \to (a_1, \ldots, a_r)^v$ means that for any r-coloring of the vertices of G there exists a monochromatic a_i-clique in G for some color $i \in \{1, \cdots, r\}$. The vertex Folkman numbers are defined as $F_v(a_1, \ldots, a_r; H) = \min\{|V(G)| : G$ is H-free and $G \to (a_1, \ldots, a_r)^v\}$, where H is a graph. Such vertex Folkman numbers have been extensively studied for $H = K_s$ with $s > \max\{a_i\}_{1 \leq i \leq r}$. If $a_i = a$ for all i, then we use notation $F_v(a^r; H) = F_v(a_1, \ldots, a_r; H)$.

Let J_k be the complete graph K_k missing one edge, i.e. $J_k = K_k - e$. In this work we focus on vertex Folkman numbers with $H = J_k$, in particular for $k = 4$ and $a_i \leq 3$. A result by Nešetřil and Rödl from 1976 implies that $F_v(3^r; J_4)$ is well defined for any $r \geq 2$. We present a new and more direct proof of this fact. The simplest but already intriguing case is that of $F_v(3, 3; J_4)$, for which we establish the upper bound of 135 by using the J_4-free process. We obtain the exact values and bounds for a few other small cases of $F_v(a_1, \ldots, a_r; J_4)$ when $a_i \leq 3$ for all $1 \leq i \leq r$, including $F_v(2, 3; J_4) = 14$, $F_v(2^4; J_4) = 15$, and $22 \leq F_v(2^5; J_4) \leq 25$. Note that $F_v(2^r; J_4)$ is the smallest number of vertices in any J_4-free graph with

chromatic number $r+1$. Most of the results were obtained with the help of computations, but some of the upper bound graphs we found are interesting by themselves.

References

[1] Zohair Raza Hassan, Yu Jiang, David Narváez, Stanisław Radziszowski, and Xiaodong Xu, On Some Generalized Vertex Folkman Numbers, to appear in *Graphs and Combinatorics*, 2023.