ISOLATION OF GRAPHS

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Since the first publication by Caro and Hansberg [4] in 2017 on isolation of graphs, many results have emerged in this area. Given a graph G and a set $\mathcal{F}$ of graphs, the $\mathcal{F}$-isolation number is the size of a smallest subset D of the vertex set of G such that $G-N[D]$ (the graph obtained from G by removing the closed neighbourhood of D) does not contain a copy of a graph in $\mathcal{F}$. Caro and Hansberg [4] established many results on $\mathcal{F}$-isolation numbers, and they also posed several problems, including that of determining the best upper bound on the cycle-isolation number (solved by Borg [2]) and on the k-clique isolation number (solved by Borg, Fenech and Kaemawichanurat [3]). Solutions will be presented together with recent bounds for the isolation of certain small graphs ([1], [6], [5]). Borg [2] showed that if $\mathcal{C}$ is the set of cycles, then the $\mathcal{C}$-isolation number of an n-vertex graph is bounded above by $n/4$ unless G is a triangle. The problem of determining a sharp upper bound on the C_k-isolation number appears very difficult. Borg's result solves the problem for k = 3. Joint work with Borg and Bartolo has yielded that the C_4-isolation number is at most $n/5$, given that G is not one of nine forbidden graphs.

References

[1] P. Borg, Isolation of connected graphs. In: arXiv (2021).

[2] P. Borg, Isolation of cycles, Graphs and Combinatorics 36 (2020), 631–637.

[3] P. Borg, K. Fenech and P. Kaemawichanurat, Isolation of k-cliques, Discrete Mathematics 343 (2020), paper 111879.

[4] Y. Caro and A. Hansberg, Partial Domination - the Isolation Number of a Graph, Filomat 31:12 (2017), 3925–3944.

[5] J. Yan, Isolation of the diamond graph, Bulletin of the Malaysian Mathematical Sciences Society (2022), in press.

[6] G. Zhang and B. Wu, $K_{1,2}$-isolation in graphs, Discrete Applied Mathematics 304 (2021), 365–374.