

ON THE TOTAL MUTUAL-VISIBILITY NUMBER OF LEXICOGRAPHIC AND CARTESIAN PRODUCTS OF GRAPHS

DOROTA KUZIAK

Universidad de Cádiz, Spain

e-mail: dorota.kuziak@uca.es

JUAN A. RODRÍGUEZ VELÁZQUEZ

Universitat Rovira i Virgili, Spain

e-mail: juanalberto.rodriguez@urv.cat

The total mutual-visibility number $\mu_t(G)$ of a connected graph G represents the cardinality of a largest set $S \subseteq V(G)$ satisfying the following property. For any two distinct vertices $x, y \in V(G)$ there is a shortest x, y -path whose interior vertices are not contained in S . The concept of total mutual-visibility number was first presented in [2], and further studied in [1, 4]. Combinatorial properties, like bounds and closed formulae, for $\mu_t(G)$ are given in this work, in which some emphasis in the case of lexicographic and Cartesian products is taken into account.

Specifically, several bounds for $\mu_t(G)$ in terms of the diameter, order and/or connected domination number of G are proved, as well as, characterizations of the graphs achieving the limit values of some of these bounds are given. Those vertices of a graph G that either belong to every total mutual-visibility set of G or does not belong to any of such sets are considered, and some consequences of these results are deduced. Moreover, it is computed the exact value of the total mutual-visibility number of lexicographic products in terms of the orders of the factors, and of the total mutual-visibility number of the first factor in the product. Finally, some bounds and closed formulae for the total mutual-visibility number of Cartesian product graphs are given.

All the results of this work are obtained in the article [3].

Acknowledgements

Dorota Kuziak was partially supported by the Spanish Ministry of Science and Innovation through the grant PID2019-105824GB-I00. Moreover, this investigation was developed while the first author (Dorota Kuziak) was making a temporary stay at the Rovira i Virgili University supported by the

program “Ayudas para la recualificación del sistema universitario español para 2021-2023, en el marco del Real Decreto 289/2021, de 20 de abril de 2021”.

References

- [1] S. Cicerone, G. Di Stefano, L. Drozdek, J. Hedzet, S. Klavzar, I. G. Yero, Variety of mutual-visibility problems in graphs, arXiv:2304.00864 [math.CO] (3 Apr 2023).
- [2] S. Cicerone, G. Di Stefano, S. Klavžar, I. G. Yero, Mutual-visibility in strong products of graphs via total mutual-visibility, arXiv:2210.07835v1 [math.CO] (14 Oct 2022).
- [3] D. Kuziak, J. A. Rodríguez-Velázquez, Total mutual-visibility in graphs with emphasis on lexicographic and Cartesian products. Manuscript, 2023.
- [4] J. Tian, S. Klavžar, Graphs with total mutual-visibility number zero and total mutual-visibility in Cartesian products, arXiv:2212.07193v1 [math.CO] (14 Dec 2022).