H-INTEGRAL AND GAUSSIAN INTEGRAL NORMAL MIXED CAYLEY GRAPHS

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A *mixed graph* G is a pair (V, E), where V is the vertex set of G, and $E \subseteq (V \times V) \setminus \{(u, u) \colon u \in V\}$ is the edge set of G such that $(u, v) \in E$ does not always imply that $(v, u) \in E$.

The (0,1)-*adjacency matrix* $[a_{uv}]$ and the *Hermitian-adjacency matrix* $[h_{uv}]$ of a mixed graph G are square matrices of order $|V|$, where

$$a_{uv} = \begin{cases} 1 & \text{if } (u,v) \in E \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } h_{uv} = \begin{cases} 1 & \text{if } (u,v) \in E \text{ and } (v,u) \in E \\ \mathbf{i} & \text{if } (u,v) \in E \text{ and } (v,u) \notin E \\ -\mathbf{i} & \text{if } (u,v) \notin E \text{ and } (v,u) \in E \\ 0 & \text{otherwise.} \end{cases}$$

Here $\mathbf{i} = \sqrt{-1}$. If all the eigenvalues of the (0,1)-adjacency matrix of a mixed graph are Gaussian integers, then the mixed graph is called Gaussian integral. If all the eigenvalues of the Hermitian-adjacency matrix of a mixed graph are integers, then the mixed graph is called H-integral.

Let Γ to be a finite group with identity element $\mathbf{1}$. For $m \geq 2$, let $G_m(1) = \{k \colon 1 \leq k \leq m-1, \gcd(k, m) = 1\}$. Define an equivalence relation $\sim$ on Γ such that $x \sim y$ if and only if $y = x^k$ for some $k \in G_m(1)$, where $m = \mathrm{ord}(x)$.

For $m \equiv 0 \pmod 4$, let $G_m^1(1) = \{k \colon k \equiv 1 \pmod 4, k \in G_m(1)\}$. Let $\Gamma(4) = \{x \in \Gamma \colon \mathrm{ord}(x) \equiv 0 \pmod 4\}$. Define an equivalence relation $\approx$ on $\Gamma(4)$ such that $x \approx y$ if and only if $y = x^k$ for some $k \in G_m^1(1)$, where $m = \mathrm{ord}(x)$.

Let $S \subseteq \Gamma \setminus \{\mathbf{1}\}$ and $\overline{S} = \{u \in S \colon u^{-1} \notin S\}$. In this talk, we show that a normal mixed Cayley graph $\mathrm{Cay}(\Gamma, S)$ is H-integral if and only if $S \setminus \overline{S}$ is a union (possibly empty) of equivalence classes of the relation $\sim$ and $\overline{S}$ is a union (possibly empty) of equivalence classes of the relation $\approx$. We further show that a normal mixed Cayley graph is H-integral if and only if the mixed graph is Gaussian integral.