PARTIAL PACKING COLORING AND QUASI PACKING COLORING OF A TRIANGULAR GRID

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The packing coloring of graphs is inspired by the problem of frequency assignment in radio networks. In this model, we assign positive integers to nodes and require that for each label (color) i and every two nodes with this label, their distance must be greater than i.

Recently, after over 20 years of intensive research, the minimal number of colors needed for packing coloring of an infinite square grid has been established to 15. Moreover, for a hexagonal grid, 7 is the minimal number of needed colors and a triangular grid is not colorable in a packing way.

Two natural questions appear: what fraction of a triangular grid can be colored in a packing model and how much do we need to weaken the condition of packing coloring to enable coloring a triangular grid with a finite number of colors?

With a partial help of the MILP solver, we proved that we can color at least 72.8% but no more than 82.2% of a triangular grid in a packing way. Moreover, we have specified the weakest (in some sense) relaxation of a condition in the definition of the packing coloring and we proved that the triangular grid is colorable in this weakened packing model with 33 colors and requires at least 9 colors.

As an open problem remains: what is the exact fraction of the grid, that is colorable in a packing model, and what is the minimum number of colors required to color this grid in a weakened packing model, or at least can we improve above bounds?

References

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