

**THE ERDOS-GYARFAS FUNCTION**  
 $F(N, 4, 5) = 5N/6 + O(N)$  — **SO GYARFAS WAS RIGHT**

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A  $(4, 5)$ -coloring of  $K_n$  is an edge-coloring of  $K_n$  where every 4-clique spans at least five colors. We show that there exist  $(4, 5)$ -colorings of  $K_n$  using  $\frac{5}{6}n + o(n)$  colors. This settles a disagreement between Erdős and Gyárfás reported in their 1997 paper. Our construction uses a randomized process which we analyze using the so-called differential equation method to establish dynamic concentration. In particular, our coloring process uses random triangle removal, a process first introduced by Bollobás and Erdős, and analyzed by Bohman, Frieze and Lubetzky. (This is a joint work with Bennett, Cushman, and Dudek.)