ON SOME VARIANTS OF DIRECTED PROPER CONNECTIONS

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An arc-colored path is properly colored if it does not contain two adjacent arcs with the same color. An arc-colored digraph D is (strong) properly connected if, between every ordered pair of vertices, there is a directed properly colored path (properly colored geodesic). The (strong) proper connection number of D is the smallest possible number of colors in a (strong) proper connected coloring. In a vertex-coloreddigraph a directed path is vertex proper if every two internal adjacent vertices differ in colors. A vertex-colored digraph D is (strong) properly vertex connected if there exists a vertex proper path (a vertex proper geodesic) between any ordered pair of vertices. The smallest number of colors that are needed in order to make D (strong) properly vertex connected is called the (strong) proper vertex connection number of D.

It is known that the proper connection number and proper vertex connection number of any strong digraph is at most 3 ([1, 3]). However, determining whether the proper connection number or proper vertex connection number of a given digraph is at most 2 is an NP-complete problem ([1]). In this talk, we will provide some sufficient conditions for a digraph to have a proper connection number or proper vertex connection number of at most 2. Additionally, we will present some basic properties of the strong proper connection number and strong proper vertex connection number.

References

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